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Abstract

A novel joint blind channel estimation and carrier offset method for code division multiple access (CDMA) communication systems is proposed. The new method combines singular value decomposition (SVD) analysis with carrier offset parameter. While existing blind methods suffer from high computational complexity because they require computation of a large SVD twice and sensitive to accurate knowledge of the noise subspace rank, the proposed method overcomes both problems by computing the SVD only once. Extensive simulations demonstrate the robustness of the proposed scheme and its performance is comparable to other existing SVD techniques with significant lower computational cost because it does not require knowledge of the rank of the noise space.

INTRODUCTION

In this paper the problem of signal recovery in code multiple access (CDMA) communication systems with unknown multiple channels and carrier offsets is considered. In CDMA systems for each user an assignment of wave signature is allocated which is used to transmit its signal. These signatures they have the orthogonality property, which allow users to simultaneously occupy the same frequency band and time frame. The receiver (mobile terminal) of the user in interest receives the signal, which is transmitted from the base station, and it must be in a position to detect the information, which is designated to him, and be able to isolate it from the rest of the signal, which represents some kind of interference.

Due to multipath effects, which are introduced by the channel, the duration of the signature is increased because of the convolution with the impulse response of the channel. This combined waveform is also known as composite signature. In order for the tracking to be possible the estimation of the unknown channel must be considered first. Apart from the channel, a second important parameter, which we must take into account, is the carrier offset. The ever-changing multipath channels and the inevitable residual carriers (in general carriers differ due to different local oscillators in the mobile terminals) present great challenges in high-speed CDMA communication systems. Furthermore, it is preferred to express a strong interest towards blind estimation techniques, since they do not require transmission of any training sequences.

The majority of joint blind channel and carrier offset estimation techniques in CDMA synchronous systems base their modeling on the method introduced in [1]. The authors in [1] consider only the multipath effect. The channel estimation technique they use is based only on the analysis of the signal and noise subspaces from the data, which the user receives. The authors in [2] consider a joint estimation of the channel and carrier offset through the calculation of eigenvalues of a polynomial problem while the authors in [3] convert the above to a general eigenvalues problem. The main characteristic of these techniques is that they are based on the signature’s samples of the user of interest which remain unaffected from the inter symbol interference (ISI), as well as that they demand to know the rank of the noise subspace. Having this information they do an SVD analysis on a large matrix in order to obtain a base of the signal and noise subspace. Then by taking advantage of the perpendicularity of the two subspaces they perform a second SVD analysis to achieve a common joint channel and carrier offset estimation. One of the drawbacks of the above methods is that the computational cost is high because it requires SVD analysis twice.

The authors in [4] looked into the problem of blind channel estimation (without considering the carrier offset parameter) by replacing the first SVD analysis for the determination of the basis of the two subspaces using a matrix raised to a power. More specifically our method follows the main line of [4] with a very essential difference in that we have incorporated in the algorithm the carrier offset parameter. We have demonstrated that our offset method performed better compared to [1], [2] and [3] at a significantly lower computational cost.

The idea we propose here overcomes the drawbacks reported in the literature. More specifically our method follows the main line of [4] with a very essential difference in that we have incorporated in the algorithm the carrier offset parameter. We have demonstrated that our offset method
performed better compared to [1], [2] and [3] at a significantly lower computational cost.

The rest of the paper is organized as follows: In Section II, we introduce the signal model for synchronous CDMA. In Section III we present our results and show a number of simulation comparisons between the proposed and existing methods. Finally Section IV concludes our work.

**MODELLING**

Our model requires no priori knowledge of the noise subspace rank. Also, it can be applied to all the samples including samples with ISI of the composite signature. The authors in [1], [2] and [3] have not included the ISI samples. In our matrix power method is not required the first step of [1], [2] and [3] which is an SVD analysis in a la [1], [2] and [3] with a simple matrix power which we will see that we can replace the first SVD analysis which is included samples with ISI of the composite signature. The (Lc + L - 1) samples from the nth symbol that are received at the receiver are given by:

\[ y(nLc + i) = s_n e^{jLc\phi_n} \sum_{k=1}^{L_c} h_k c_{i-k+1} e^{j\phi_k}, i = 1, ..., L_c + L - 1 \]

\[ \Rightarrow \begin{bmatrix} y(nL_c + 1) \\ y(nL_c + 2) \\ \vdots \\ y(nL_c + [L_c + L - 1]) \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{L_c+L-1} \end{bmatrix} S_n e^{jLc\phi_n} \]

Where the samples of the composite signature are as follows:

\[ \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{L_c+L-1} \end{bmatrix} \begin{bmatrix} e^{j\phi_1} \\ e^{j\phi_2} \\ \vdots \\ e^{j(L_c+L-1)\phi} \end{bmatrix} \begin{bmatrix} c_1 \\ c_1 \\ \vdots \\ c_1 \end{bmatrix} \begin{bmatrix} h_1 \\ \vdots \\ h_{L_c+L-1} \end{bmatrix} \]

\[ \Rightarrow W_{(L_c+L-1)x1} = \sum_{i=1}^{L_c+L-1} \sum_{k=1}^{L_c} h_k e^{j(k-1)\phi} \begin{bmatrix} c_1 \\ c_1 \\ \vdots \\ c_1 \end{bmatrix} \begin{bmatrix} h_1 \\ \vdots \\ h_{L_c+L-1} \end{bmatrix} \]

We now focus now to the user of interest (Zi=Z, Ci=C, Hi=h). We assume that the user obtains, with its receiver N vectors of data, each one of these vectors constitute the overlapping of the symbols of the P users. Taking into account the noise vectors which lay on the matrix N_{(L_c+L-1)xN}, the N vectors which received they are given by:

\[ X_{(L_c+L-1)xN} = W_{(L_c+L-1)xP} S_{P\times N} + N_{(L_c+L-1)xN} \]

Performing an SVD analysis on the autocorrelation matrix of (3), we end up with:

\[ R = \begin{bmatrix} U_1 & \ldots & U_P \end{bmatrix} \begin{bmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \sigma_P^2 \end{bmatrix} \begin{bmatrix} U_1^\dagger \\ \ldots \\ U_P^\dagger \end{bmatrix} \]

Exploiting the orthogonality between signal and noise subspace we have:

\[ U_n^H U_n = 0 \Rightarrow U_n^H Z C h = 0 \]

\[ \Rightarrow (U_n^H ZC)^H (U_n^H ZC) h = 0 \]

\[ \Rightarrow (C^H Z U_n U_n^H ZC) h = 0 \]

\[ U_n U_n^H \] constitutes the orthogonal projection matrix, to the noise subspace, therefore from equation (4) we have:

**TABLE I**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Sn</td>
<td>The sequence of the information symbols</td>
</tr>
<tr>
<td>Lc</td>
<td>Number of chips, spreading gain</td>
</tr>
<tr>
<td>c</td>
<td>User’s code</td>
</tr>
<tr>
<td>N</td>
<td>Number of received vectors</td>
</tr>
<tr>
<td>Rxz</td>
<td>Autocorrelation matrix</td>
</tr>
<tr>
<td>φ</td>
<td>Phase of the carrier</td>
</tr>
<tr>
<td>L</td>
<td>Length of the channel h</td>
</tr>
<tr>
<td>σ</td>
<td>The smallest eigenvalues</td>
</tr>
<tr>
<td>P</td>
<td>Number of users</td>
</tr>
</tbody>
</table>

**NOTATIONS**
From equation (5) it can be observed that by raising the inverse autocorrelation matrix to a power we can approach the projection matrix $U_nU_n^H$, which in practice does not need to be higher than three ($k=3$). So from (4) and (5) we get:

$$Q_{LxL} h = 0$$

Finally we end up to the known problem of eigenvalues but without performing an SVD analysis to the autocorrelation matrix in order to have, an estimation of the base for the noise subspace.

**SIMULATION RESULTS**

The steps required to perform the joint channel estimation and carrier offset power method is:

1. Calculate the autocorrelation matrix raised to the power $k$ where $k=1, 2$ and $3$.
2. Sampling of $\phi$ in space $[-0.1, 0.1]$, where it is assumed to fluctuate, the smallest values of $\phi$ are stored in matrix $Q$. The estimation of the carrier offset it would be that $\phi$ which gives the smallest eigenvalue. The estimation of the channel is the eigenvector which corresponds to that eigenvalue.

For the channel we assume that the impulse response has duration of $LT_c$. By sampling the channel with sampling rate $R_c=1/T_c$, we take $L$ samples and we therefore can model that channel as a FIR (Finite Impulse Response) filter with coefficients the $L$ samples:

$$h = (h_1, \ldots, h_L)^T$$

Then we calculate the MSE (mean square average) error of the estimation of the carrier offset and of the channel for an (signal to noise ratio) SNR varying from 0 to 30 dB ($x$-axis of figure 2) using $N=1000$ symbols.

Next we will simulate the power method taken together with a subspace analysis method. Then the solution of a polynomial eigenvalue problem with SNR = 10dB in conjunction with the number of symbols which has been received.

The autocorrelation matrix is being estimated in advance as follows:
\[ R_{xx}(n) = \lambda R_{xx}(n-1) + x(n)x^H(n) \]  (7)

\begin{array}{c}
\phi : 0.0122 \\
\phi : 0.0126 \\
h = 0.4070 \ 0.8150 \ 0.4070 \\
\bar{h} = -0.4081 \ -0.8181 \ -0.4050
\end{array}

Figure 1. The smallest eigenvalues in conjunction with samples (phi) \( \phi \)-Hard channel.

Then we calculate the MSE (mean square average) error of the estimation of the carrier offset and of the channel for an (signal to noise ratio) for various SNR values using \( N=100 \), \( N=1000 \) and \( N=10000 \) symbols for the generation of the autocorrelation matrix. We observe that only when the arithmetic average is big enough (\( N>> \)) the rule of big numbers applies and then we have better results for \( k>1 \).

![Figure 2 (a &b). Formulation of autocorrelation matrix, N=100 symbols against SNR](image)
Figure 3 (a & b). Formulation of autocorrelation matrix, N=1000 symbols against SNR

Concluding we simulate the power raising method with that one of the subspace decomposition and then to the salvation of a polynomial eigen value problem in environment with SNR=20dB, in conjunction with the number of symbols being received.

With $\lambda = 0.997$ to correspond to a $\frac{1}{1-\lambda} = 333.333$ samples window, while the inverse matrix of autocorrelation which is used in the power method is:
\[
R_{xx}^{-1}(n) = \frac{1}{\lambda} \left[ R_{xx}^{-1}(n-1) - \frac{R_{xx}^{-1}(n-1)x(n)x^H(n)R_{xx}^{-1}(n-1)}{\lambda + x^H(n)R_{xx}^{-1}(n-1)x(n)} \right]
\]  

(8)

With initial value: \( R_{xx}^{-1}(0) = \delta I \) where \( \delta = \frac{100}{\sigma_n^2} \)

In Figures 5 and 6 it can be observed, that the power method converges faster to lower estimation levels compared to Subspace Decomposition (SD) methods. After the channel change or the carrier offset changes at 2000 symbol, for \( k=1 \) the method downgrades faster the estimation error requiring approximately 400 symbols, for \( k=2 \) approx. 500 and for \( k=3 \) approx. 700. It can be observed from the simulation windows that the raised power method is better compared to the SD method. The x-axis of the simulation graphs is the number of transmitted symbols ranging from 0 to 4000 symbols, and the y-axis is the carrier offset estimation in dB for figure 5 and channel estimation MSE for figure 6. In order to illustrate the performance of our method in this paper we use BPSK modulation with number of users \( P=10 \), spreading gain \( L_s=32 \), and \( N=100 \) the received number of data vectors at the receiver. The SNR is set at 20dB.

In Figure 3 we can observe the behavior of our method using \( k=1, 2 \) and 3 in comparison with SD. We can clearly see that after a small number of transmitted symbols (100) our method has lower carrier offset estimation error in dB than the SD method. Moreover, we can observe that when we have a change of the serving channel chosen to be at symbol 2000, our method performs better immediately after the change since the carrier offset estimation error is at lower levels for the power method in comparison with the SD method. In figure 6, we trigger a change in the carrier offset \( (\phi) \), the proposed method performs better again since the channel estimation error is at lower levels before and after the sudden change in phase \( (\phi) \) occurred at transmitted symbol 2000.

**Conclusion**

In this paper we presented a novel method for joint channel and carrier offset estimation for CDMA communication systems. Our method is based on a two-step methodology including the offset carrier parameter in the power method presented in [4] which has the advantage of reducing a two-step SVD analysis to a single step. Also, the performance of this method is independent of the knowledge of the signal subspace rank whereas the approaches in [1], [2], [3] are sensitive to correct knowledge of this parameter. As a result our technique performed better compared to other existing techniques at a significantly lower computational cost.

**References**


Biographies

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