On commodity taxation in vertically differentiated markets

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Abstract

We examine the impact of commodity taxation on vertically differentiated product markets when entry is allowed. We show that an ad valorem tax may have a dramatic effect on market structure by inducing the entry of a large number of firms in what was previously a natural monopoly. The producers of high quality products reduce market share after an increase in their unit production cost, leaving more room for lower quality products. While within a given market structure aggregate quality decreases monotonically with the tax rate, quality jumps upwards at tax rates that cause a change in market structure. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Commodity taxation has been a primary focus in the literature of public finance. Many studies examine the impact of excise or \textit{ad valorem} taxes on prices, on
output of different commodities and on welfare. The majority of these studies are based on a competitive framework (Auerbach, 1985; Kotlikoff and Summers, 1987) provide excellent surveys of this literature.) Monopolies and homogeneous Cournot oligopolies have also been studied, but to a lesser extent. Differentiated product markets have until recently received little attention, mostly through representative consumer models.¹

In an important contribution, Crémer and Thisse (1994a) introduced ad valorem taxes in a vertically differentiated duopoly. The novelty of their approach consists not only in studying the direct effect of taxes on prices, but also in considering their indirect effect, through their impact on firms’ quality choice. They show that an ad valorem tax brings qualities closer together and results in intensified competition that lowers the prices of both product variants. More surprisingly, they show that a small uniform ad valorem tax is always welfare improving.

The analysis of Crémer and Thisse is limited by the fact that market structure is exogenously given. In this paper, we allow for a large number of potential entrants and examine how the tax affects market structure, as well as how it affects the specification of the products available in a vertically differentiated market. While we focus on a taxation issue, our analysis has broader IO implications in that it provides comparative static properties with respect to shifts in the marginal cost function in vertical differentiation models.

We consider a good differentiated by quality, which implies consumer unanimity over product ranking. We assume that this unanimity not only holds at equal prices (as in Crémer and Thisse), but is also maintained if all the varieties are priced at their respective marginal cost. In this situation there is an upper bound, independent of the fixed cost, to the number of active firms in equilibrium.² We show that an ad valorem tax may have a radical impact on market structure. By increasing the steepness of the marginal cost schedule, such tax can either raise the upper bound of the number of firms or even eliminate it completely. In the former case, a less concentrated oligopoly may emerge in the after-tax situation. In the latter case a marginal increase in the tax rate may cause an abrupt change in market structure, inducing, for instance, the entry of a large number of single product firms in what was a natural monopoly in the pre-tax situation.

The endogeneity of market structure has major implications for the tax design. When consumer unanimity over product ranking holds at prices reflecting marginal cost, social optimality requires that everybody consumes the highest feasible quality. In the standard case of homogeneous products total quantity decreases monotonically with the tax rate. Likewise, in the case of vertically differentiated products total quality decreases monotonically with the tax rate within a given market structure. This suggests that quality underprovision under monopoly

¹ For more detailed references, see Crémer and Thisse (1994a).
² See Gabszewicz and Thisse (1979); Shaked and Sutton (1982) and Shaked and Sutton (1983)
requires the use of a negative tax, i.e., a subsidy. However, marginal increases in the tax rate that alter market structure also cause discontinuous increases in aggregate quality. This in turn implies that a carefully selected tax rate can achieve any aggregate quality level yielded by a subsidy. This point is particularly important when quality is associated to negative externalities: the regulator must now trade off marginal consumption benefits from changes in quality against marginal external damages. Our analysis suggests that particular care is required in the selection of the optimal tax rate because of the non-monotonicity of the aggregate quality schedule with respect to changes in the tax rate.

The rest of the paper is organized as follows: Section 2 describes the model; Section 3 reviews the finiteness property and the effect of an ad valorem tax on market structure; Section 4 describes quality under monopoly as well as with an infinite number of firms in the market. Section 5 contains the concluding remarks.

2. The model

Consider a number of single product firms producing substitute goods. Their respective products are labelled by an index \( i, i = 1, \ldots, n \), and their respective prices denoted by \( p_i \).

On the demand side, we assume a continuum of consumers with identical tastes uniformly distributed according to their incomes \( t \) over the interval \([a, b] \), \( 0 < a < b \). For simplicity we assume the density of the income distribution equals one. Consumers have inelastic demands, purchasing either one unit of product or none at all. The utility of a consumer of type \( t \) derived from the consumption of one unit of product \( i \) is

\[
U(t, u_i) = u_i (t - p_i),
\]

with \( u_1 > u_2 > \cdots > u_n \). It is obvious from Eq. (1) that, at equal prices, consumers are unanimous over product ranking. Hence, we can define \( u_i \) as the quality level of product \( i, i = 1, \ldots, n \). The non-purchase option is represented by a reservation quality \( u_R < u_n \) available at zero price, so \( U(t, u_R) = u_R t \). We define \( r_i \) as an index of relative qualities, \( r_i = u_i / (u_i - u_{i+1}), i = 1, 2, \ldots, n \) and \( r_n = u_n / (u_n - u_R) \). Then, we define \( t_i \) as the income level of the consumer who is indifferent between qualities \( u_i \) and \( u_{i+1} \) at their respective prices, i.e.

\[
u_i (t_i - p_i) = u_{i+1} (t_i - p_{i+1}), \quad i = 1, \ldots, n,
\]

In Crémer and Thisse (1994a) a tax within a market structure (duopoly) decreases quality but contrary to our case increases welfare. This is due to the fact that at prices reflecting marginal cost consumer ranking of qualities is not unanimous, thus giving rise to the possibility of quality overprovision in the pre-tax equilibrium.
from which we obtain,
\[ t_i = p_i r_i - p_{i+1}(r_i - 1), \quad i = 1, 2, \ldots, n - 1, \]  
(2)

and
\[ t_n = p_n r_n. \]  
(3)

On the supply side we assume that firms can choose their product from a range of technologically feasible qualities \([u_k, \bar{u}]\). In order to simplify the exposition, we define \( z = 2\bar{u} - u_k \), an index related to the width of this range. We assume the cost of \( u_k \) equals zero, \( c(u_k) = 0 \), and write the variable unitary cost function of quality \( i \) as,
\[ c(u_i) = \gamma(u_i - u_k), \quad \forall u_i \in (u_k, \bar{u}). \]  
(4)

Note that the above cost function exhibits increasing average cost of quality, \( c(u_i)/u_i \), and constant returns to scale in quantity. In order to start production, a fixed cost \( F \) independent of quality is also required. The amount of \( F \) is not recoverable if the firm ceases to produce.

We consider firms playing a three-stage game with an infinite number of potential players. Entry decisions are taken sequentially during the first stage. Qualities are chosen simultaneously at the second stage by firms already in the market. Finally, at the third stage of the game, firms choose their prices simultaneously. The postulated solution concept is that of subgame perfectness. The model is solved backwards, starting from the price stage.

3. The finiteness property

A crucial characteristic of vertically differentiated markets is that there may be an upper bound to the number of firms that are able to survive in a Bertrand–Nash equilibrium. In the following, Lemma 1, adapted from Shaked and Sutton (1983), determines the conditions for the presence of this upper bound while Lemma 2 determines its value.

**Lemma 1.** Define \( \gamma = a/z \), where \( z = 2\bar{u} - u_k \). i) For \( \gamma < \gamma_f \) there is an upper bound to the number of active firms in a Nash equilibrium (Finiteness Property). This bound depends on the width of the income distribution and the characteristics of the cost function, and is unrelated to the level of the fixed cost \( F \). ii) For \( \gamma > \gamma_f \) the equilibrium number of firms will be arbitrarily large for sufficiently small values of \( F \).

**Proof.** It is well known that a necessary and sufficient condition for the finiteness property to hold is that consumer unanimity over product ranking is maintained.
when all products are priced at average cost. A sufficient condition for this is that for any consumer with income $t$, $\partial U(t-c(u),u)/\partial u > 0$, $\forall u$. Given our cost-preference structure this is equivalent to

$$t - 2\gamma u + \gamma u_\gamma > 0, \quad \forall t \in [a, b], \quad \text{and } \forall u \in [u_\gamma, \bar{u}].$$

(5)

Since the LHS of Eq. (5) is increasing in $t$ and decreasing in $u$, the necessary and sufficient condition for Eq. (5) to hold is obtained by substituting in its LHS the values of $a$ and $\bar{u}$ for $t$ and $u$ respectively. The condition stated in the lemma is obtained straightforwardly. This proves (i).

Define $\bar{U}' = \partial U(t-c(u),u)/\partial u|_{u=a}$ and $\bar{t}$ the value of $t$ such that $\bar{U}'(\bar{t}) = 0$. The LHS of Eq. (5) is monotonically increasing in $t$ and $2\bar{u} > u_\gamma$. When $\bar{t} > a$, for all consumers with income $t \in [a, \bar{t})$, $\bar{U}' < 0$. Hence, for those consumers there exists a $u^*(t) < \bar{u}$ setting the LHS of Eq. (5) equal zero. The $u^*(t)$ schedule is continuous and monotonically increasing, so any quality between $u^*(a)$ and $u^*(\min[\bar{b}, \bar{t}])$ can find a customer at marginal cost. It follows that as $F$ tends to zero an infinite number of products will enter the market. □

Lemma 1 divides the set of values of $\gamma$ in two subsets according to whether $\gamma$ is smaller or greater than a critical value $\gamma_\gamma$. The main difference between the two situations is that in case (i) $c(u)$ increases more slowly relative to consumer willingness to pay, whereas in case (ii), $c(u)$ increases more quickly relative to consumer willingness to pay. As a result, in case (ii) when all products are offered at unit variable cost, consumer preferences will be split between expensive high quality products and inexpensive low quality ones. The situation resembles the familiar location paradigm (see for instance Hotelling (1929), and d’Aspremont et al. (1979)) where a decrease in $F$ results in new firms locating between any two established products; as $F$ approaches zero, so does the value of the $H$-index for this market. On the other hand, when $\gamma < \gamma_\gamma$, (case i)) all consumers’ willingness to pay for quality improvements is higher than the cost increments necessary to provide these improvements. As a result, consumer unanimity over product ranking is maintained when all products are available at unit variable cost. Hence, high quality firms maintain over their lower quality rivals a product differentiation advantage despite cost differences. In this situation, market fragmentation reaches a bound even if $F$ approaches zero. The driving force behind this result is that competition between active firms drives their prices down to a level where every consumer prefers to buy one of the “surviving” products at its equilibrium price or make no purchase, rather than buying any of the excluded products at any price.

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1. I.e. at prices reflecting average cost no consumer finds a quality that yields an interior solution to his/her utility maximization problem.
2. Note that when $a < \bar{t} < b$ all consumers with $t = [\bar{t}, b]$ will purchase $\bar{u}$ while when $\bar{t} > b$ everybody purchases a different quality and nobody buys $\bar{u}$. See the next section for more details.
that would cover their unit variable cost. It must be stressed that the existence and
value of the upper bound on the number of firms stem from the pricing stage of the
game and are therefore independent of quality choices of the surviving products,
as can be seen from the expression for $\gamma_f$.

Assuming the presence of the finiteness property, the following lemma
determines the largest number of firms that can be active in the Bertrand–Nash
equilibrium of the game.

**Lemma 2.** When $\gamma < \gamma_f$ an upper bound to the number of firms is determined by
the following expression

$$n < 1 + \frac{b - a}{a - \gamma \epsilon}.$$  (6)

**Proof.** Assume there are $n$ active firms. Firm $k$ chooses $p_k$ to maximize
$R_k = (p_k - c_k)(t_{k-1} - t_k)$. Setting $\partial R_k / \partial t_k = 0$ yields

$$M_k = t_{k-1} - t_k = (p_k - c_k)u_k\frac{u_{k-1} - u_{k+1}}{(u_k - u_{k+1})(u_{k-1} - u_k)}, \quad k = 2, \ldots, n - 1,$$

which in turn implies

$$M_k > \frac{u_k p_k - u_{k+1} p_{k+1}}{u_k - u_{k+1}} - \frac{u_k e_k - u_{k+1} e_{k+1}}{u_k - u_{k+1}} = t_k - \gamma(u_k + u_{k+1} - u_k),$$

$$> a - \gamma \epsilon.$$  

Similarly, it can be shown that $M_1 = b - t_1 > a - \gamma \epsilon$. Thus, $b - a > \sum_{k=1}^{n-1} M_k > (n - 1)[a - \gamma(2a - u_k)]$ which, after some straightforward rearrangement, yields the
upper bound to the number of active firms as stated in expression Eq. (6). \qed

Notice that the denominator of the fraction on the RHS of Eq. (6) is always
positive since $\gamma < \gamma_f$. From Eq. (6) it is obvious that the upper bound to
the number of firms is positively related to the width of the income distribution,
$(b - a)$, the cost parameter $\gamma$, and the index $\epsilon$. In order to understand the intuition
of the first of these influences, let us start from a situation where $a = b$. In such a
case there is no room for differentiation: the high quality producer will supply $u_\bar{a}$.
As we reduce $a$, the high quality producer needs to reduce his price in order to
cover the entire market. When $a$ is at a sufficient distance from $b$, the high quality
firm prefers to forego some customers rather than further decrease its price. At that
point, if the fixed cost is not too high, a second firm may enter the market and the
same process will be repeated as long as the finiteness property holds. Increases in

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The proof follows the one in Anderson et al. (1992, p. 310). The latter examines the upper bound to
the number of firms in the case of linear utility function and zero marginal cost.
\( \gamma \) reduce the higher qualities’ optimal market share leaving more room for the entry of lower qualities. Finally, as \( z \) increases it becomes easier for a lower quality firm to distance its product from higher qualities. This makes puppy-dog entry easier.

It must be stressed that the RHS of Eq. (6) being greater than \( n \) yields a necessary but not sufficient condition for the presence of \( n \) active firms in the market. A sufficient condition cannot be obtained without reference to the equilibrium qualities. These are extremely difficult to obtain analytically even in the simple case of natural duopoly. Obviously, the non satisfaction of the necessary condition for duopoly is sufficient to guarantee the presence of a single firm in the market. When \((b-a)/(a-\gamma z)\leq 1\) the market is a natural monopoly. We define as \( \gamma_1 \) the value of \( \gamma \) that establishes this expression with equality. Hereafter we focus our analysis to the case in which the equilibrium market structure in the pre-tax situation is a natural monopoly.

Let \( \gamma_0<\gamma_1<\gamma_2 \) be the before-tax value of \( \gamma \) and assume the introduction of an ad valorem tax at the rate \( \varphi \). After defining \( \varphi = 1/(1-\varphi)>1 \), the after-tax profits of firm \( k \) are \( R_k = (1/\varphi)(p_k-\tau c_k)(t_{k-1}-t_k) \). It is clear that an increase in the tax rate is equivalent to an increase in marginal cost. Define \( \gamma_2 \) such that:

\[
b - 2a = - [\gamma_2 u_k(8b + \sqrt{\gamma_2 u_k})]/4.
\]

**Proposition 1.** As \( F \to 0 \), i) a “small” tax rate defined as \( \tau \leq \gamma_1/\gamma_0 \) leaves the market structure unchanged, ii) a “medium” tax rate \( \gamma_1/\gamma_0 < \tau \leq \gamma_2/\gamma_0 \) will induce an oligopoly, and iii) a “large” tax rate \( \tau > \gamma_2/\gamma_0 \) may induce the entry of an arbitrarily large number of firms.

**Proof.** The first and third part of the proposition are straightforward corollaries of lemmata 2 and 1 respectively. When gamma \( \gamma < \gamma_1 \) a sufficient condition to have more than one firm in the market is that the optimal quality-price configuration of the top quality product induces partial market coverage. To see this, notice that the excluded consumers prefer any \( u > u_k \) priced at marginal cost to \( u_k \) for free. One needs therefore to maximize

\[
R_m = (p_m - c_m)(b - t_m)
\]
with respect to \((p_m, u_m)\), compute the optimal value of \(t_m^*\) and require that the latter be greater than \(a\). The above maximization problem yields

\[
u_m^* = \gamma u_R + \sqrt{\gamma u_R (8b + \gamma u_R)} / (4\gamma) \quad p_m^* = 4b - \gamma u_R - \sqrt{\gamma u_R (8b + \gamma u_R)} / 8
\]

(8)

from which we obtain that \(t_m^* = p_m^*, \gamma > a\) implies

\[
b - 2a > - \gamma u_m^* = - [\gamma u_R + \sqrt{\gamma u_R (8b + \gamma u_R)}] / 4
\]

(9)

When Eq. (9) does not hold the monopolist covers the entire market. Obviously, the RHS of Eq. (9) is decreasing in \(\gamma\). Denote as \(\zeta_2\) the value of \(\gamma\) for which Eq. (9) holds with equality and assume that \(\zeta_2 < \gamma\). Starting from a \(\zeta_2 < \gamma\) a tax rate \(\tau \in (\zeta_2 / \zeta_0, \gamma / \gamma_0)\) will induce an oligopoly while a \(\tau < \gamma / \gamma_0\) will let market structure remain unchanged. □

Setting \(\gamma = 0\), the condition in Eq. (9) reduces to the well-known condition \(b > 2a\) derived in Shaked and Sutton (1982) for the case that marginal cost is independent of quality. Condition Eq. (9) is sufficient for the presence of a second firm in the market. Setting the RHS of Eq. (6) equal to two, one can define \(\gamma_5\) such that when \(\gamma_0 \tau \in (\zeta_2, \zeta_1)\) the market will be a natural duopoly. When \(\gamma_0 \tau \in (\zeta_1, \zeta_1)\) only a detailed analysis of the situation can reveal whether the tax transforms the market to a duopoly or leaves market structure unaltered.

We were not able to generalize Eq. (9) as to find a sufficient condition for the presence of \(n\) firms. We believe, however, that as the tax steepens the \(c(u)\) schedule a new firm will enter periodically before the finiteness condition breaks up completely. Our belief is based on the conjecture that as \(c(u)\) becomes steeper, the equilibrium market shares of the \(n - 1\) top qualities will be reduced.\(^{12}\) On the other hand, as long as the market is covered the \(n\)-th firm will always adjust its price in

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\(^{11}\) The maximization with respect to \(u_m\) yields three roots one of which is always negative, the second being the one in the text and the third being \(\hat{u} = b / \gamma\). It can be easily shown that \(\hat{u} > u_m^*\) and that \(\tau^*(\hat{u}) = b\), therefore no \(u > \hat{u}\) is admissible. Moreover, \(R'(p(u_m^*), u_m^*)\) is negative (positive) around \(u_m^* \) (\(\hat{u}\)).

\(^{12}\) With fixed qualities, an increase in \(\gamma\) increases the price reaction functions of all the \(n - 1\) higher qualities. Because prices are strategic complements, this induces higher prices and a higher \(t_{n-1}\). On the other hand, increases in the marginal cost of quality imply that, for given rival qualities, each firm wishes to reduce its own quality. Thus, quality reaction functions move downwards following an increase in \(\gamma\). Since qualities are also strategic complements, equilibrium qualities will be lowered which also tends to increase \(t_{n-1}\), (i.e., market coverage will be reduced for given prices). While the reduction in qualities tends to lower prices and therefore countervail the upward movement of \(t_{n-1}\) after an increase in \(\gamma\) the equilibrium value of marginal cost should not be lower than before. It follows that the new equilibrium with higher \(\gamma\) implies both lower qualities and higher prices (albeit lower than what would obtain with fixed qualities) for the \(n - 1\) top qualities.
order to set $t_n^* = a$. As $\gamma$ increases and $t_{n-1}$ moves upward, firm $n$ has more of an incentive to increase its price and ignore some customers in the neighbourhood of $a$, which makes room for the entry of an additional firm.

By steepening the $c(u)$ schedule, an *ad valorem* tax reduces the product differentiation advantage of the higher qualities. When the $c(u)$ schedule becomes sufficiently steep ($\gamma > \gamma_1$) this advantage is completely eliminated: while all consumers still prefer higher over lower qualities, unanimity disappears when all products are offered at their after-tax marginal cost. Each consumer now has her preferred type between inexpensive-low quality and expensive-high quality products. No quality corresponding to a consumer’s preferred type can be priced out of the market unless some higher quality is priced below marginal cost. When $F \to 0$, a whole spectrum of qualities will be available. It must be noted that this increase in the number of available products relative to the pre-tax situation does not necessarily make consumers better off, even if all the products in this extreme case of $F \to 0$ are priced at marginal cost. If every consumer finds now “exactly what she wants”—i.e., she has an interior rather than a corner solution to her utility maximization problem—it is because the steepening of the $c(u)$ schedule has made higher qualities relatively less attractive.

4. Quality stage

The results of the previous section show the dramatic impact an *ad valorem* tax may have on the number of single quality firms. In this section we examine the quality choices of the active firms and the resulting level of aggregate quality. As the analytical solutions for the optimal qualities are not tractable even in the simple case of natural duopoly, we rule out the possibility of a natural oligopoly and “medium” range taxes. For this, we assume that $\gamma_0 < \gamma_1 < \gamma_f$ so the equilibrium market structure in the pre-tax situation as well as in all cases in which the finiteness property holds is a natural monopoly.

Lemmas 3 and 4 analyse quality selection within the monopolistic and competitive market structure respectively, while Proposition 2 describes the relation between the tax rate and the aggregate quality. To keep matters simple, in all the situations considered below it is assumed that no consumer chooses the reservation quality in equilibrium (covered market). This assumption keeps total quantity constant and allows us to focus on firms’ quality choices and the aggregate quality consumed. Moreover, it implies that any price change constitutes a transfer between producers, consumers and the government.

Increases in quality are *ceteris paribus* related positively to consumer welfare through the utility function in Eq. (1). When $\gamma_0 < \gamma_1$ social welfare maximization requires that everyone consumes $\bar{u}$. This is so because, at that quality level, the marginal utility of quality improvements exceeds their marginal cost for all
consumers. The following result shows that the monopolist underprovides quality unless $\gamma$ is very small.

**Lemma 3.** If $\gamma < \gamma_f$ and $F$ is not too large, the aggregate level of quality consumed, $A(\gamma)$, is equal to $(b-a)u^M(\gamma)$ where $u^M(\gamma) = \min(\bar{u}, (au_r \gamma^{-1})^{1/2})$. Unless $\gamma < \bar{u}/(au_r)$, the monopolist underprovides quality.

**Proof.** Under our assumptions $\gamma < \gamma_f$ implies a natural monopoly. We first show that, for any quality choice, the monopolist will always price his product as to cover the entire market ($p_1 \leq a/r_1$) while leaving no positive surplus to the poorest consumer ($p_1 \geq a/r_1$), hence $p_1 = a/r_1$.

The monopolist will never set $p_1 < a/r_1$ since this would imply $t_1 < a$. In this case a small price increase would not affect sales and therefore such an increase would increase profits. If $p_1 > a/r_1$, the monopolist would leave part of the market uncovered. This is ruled out by the fact that $\gamma_0 < \gamma_1 < \gamma_f$ where $\gamma_1$ is the value of $\gamma$ that satisfies Eq. (9) with equality. Thus, the monopolist just covers the market, $t_1 = a$, and his optimal pricing rule is $p_1 = a/r_1$.

Substituting the optimal $p_1$ and $t_1$ into the monopolist’s profit function Eq. (7) and taking the derivative with respect to quality we find the interior solution $u^M(\gamma) = (au_r \gamma^{-1})^{1/2}$. This expression, multiplied by $(b-a)$ to account for the fact that all the consumers buy one unit of the product, yields the expression in the lemma. There exists a value $\gamma_{min}$ such that for $\gamma < \gamma_{min}$, $u^M(\gamma) > \bar{u}$ in which case the monopolist chooses $\bar{u}$ as a corner solution. \(\Box\)

The socially optimal level of aggregate quality is $(b-a)\bar{u}$ and therefore, for all $\gamma > \gamma_{min}$, the monopolist underprovides quality. The above lemma also implies that, except for corner solutions and as long as the monopoly structure remains unaffected, increases in the tax rate lower quality. Thus, starting with a $\gamma$ such that, $\gamma_f > \gamma > \gamma_{min}$, an ad valorem tax increases the market distortion, i.e. the underprovision of quality. This result is similar to the one derived in the case of imperfect markets for homogeneous goods. In that case, a tax results in the underprovision of quantity.

The next result examines aggregate quality for values of $\gamma > \gamma_f$. Since the finiteness property does not hold for these values, small levels of $F$ induce the entry of a large number of firms and the consumption of many qualities in equilibrium.

**Lemma 4.** When $\gamma > \gamma_f$, as $F \to 0$, $A(\gamma)$ tends to i) $(\gamma z - a)(\gamma z + a + 2u_r \gamma)/4 \gamma + (b - \gamma z)\bar{u}$ for $\gamma_f \leq \gamma < \gamma_0$; ii) $(b-a)(b+a+2u_r \gamma)/4 \gamma$ for $\gamma > \gamma_0$, where $\gamma_0 = b/z > \gamma_f$.

13 Recall that at prices reflecting average variable cost all consumers would unanimously choose $\bar{u}$. 

Proof. Notice first that as \( F \to 0 \) the available qualities tend to a continuum and this proximity drives prices down to marginal cost. Considering \( p(u) = c(u) = \gamma(u - u_R) \), and maximizing consumers’ utility Eq. (1) with respect to quality we obtain

\[
\begin{align*}
 u^*(t) &= \min \left\{ \frac{t + u_R \gamma}{2\gamma}, \tilde{u} \right\} .
\end{align*}
\]  

(10)

Setting the first term in brackets equal to \( \tilde{u} \) and solving for \( t \) we obtain \( \tilde{t} = \gamma \), the income level of the consumer who chooses \( \tilde{u} \) as an interior maximum. If \( \gamma \leq \tilde{t} \) then consumers with income \( t \in [a, \gamma) \) choose a quality lower than \( \tilde{u} \) according to Eq. (10), while those with \( t \in [\gamma, b] \) choose \( \tilde{u} \). If \( \gamma > b \) all consumers have an interior solution in their quality choice problem. Integrating optimal quality choices over incomes we obtain

\[
A(\gamma) = 
\begin{cases} 
\int_{a}^{\gamma} \left( \frac{t + u_R \gamma}{2\gamma} \right) dt + (b - \gamma) \tilde{u} = (b - \gamma) \tilde{u} + \frac{(\gamma - a)(\gamma + a + 2u_R \gamma)}{4\gamma} & \text{for } \gamma < \gamma \leq \gamma_b \\
\int_{a}^{b} \left( \frac{t + u_R \gamma}{2\gamma} \right) dt = \frac{(b - a)(b + a + 2u_R \gamma)}{4\gamma} & \text{for } \gamma \geq \gamma_b .
\end{cases}
\]

(11)

The values of \( A(\gamma) \) in Eq. (11) are those referred to in Lemma 4. □

As shown in Lemma 4, for values of \( \gamma \geq \gamma_f \) no consumers with differing incomes choose the same quality, while for \( \gamma_f < \gamma \leq \gamma_b \) some consumers will be bunched into purchasing the top quality. Dividing both the numerator and denominator of the RHS of the second expression in Eq. (11) by two, we see that when \( \gamma \geq \gamma_f \), \( A(\gamma) \) increases proportionately with mean-preserving increases of the width of the income distribution.

Using lemmata 3 and 4 we can see that, within any given market structure, aggregate quality is monotonically decreasing in \( \gamma \), \( \partial A(\gamma)/\partial \gamma < 0 \). Moreover, \( \partial^2 A(\gamma)/(\partial \gamma)^2 > 0 \) for \( \gamma < \gamma_f \), as well as for \( \gamma > \gamma_b \) while \( \partial^2 A(\gamma)/(\partial \gamma)^2 < 0 \) for \( \gamma_f \leq \gamma \leq \gamma_b \). The next result investigates the impact of an \textit{ad valorem} tax when the endogeneity of the market structure is taken into account.

**Proposition 2.** With endogenous market structure aggregate quality does not decrease monotonically with the tax rate.

Proof. Simply take the limits of \( A(\gamma) \) as \( \gamma \to \gamma_f^+ \) from below and \( \gamma \to \gamma_f^- \) from above. The former is obtained from Lemma 3 \( \operatorname{lim}_{\gamma \to \gamma_f^+} A(\gamma) = \lim_{\gamma \to \gamma_f^-} \frac{(b - a)(u_R \gamma^{-1})^{1/2} = (b - a)^{1/2} (u_R)^{1/2} }{1} \) and is obviously smaller than the latter, \( \operatorname{lim}_{\gamma \to \gamma_f^-} A(\gamma) = (b - a)\tilde{u} \), obtained from the first expression in Eq. (11). □
The above result shows that, starting from a natural monopoly situation, a large tax (according to the terminology of Proposition 1) may increase aggregate quality. This result is illustrated in Fig. 1. On the vertical axis we measure aggregate quality consumed, while on the horizontal we measure the parameter $g$. The curve BC represents aggregate quality for values of $g$ for which only one quality is offered. From Lemma 3 above, the curve BC is decreasing and convex in $g$. At the value $g^*$, the finiteness property breaks down and new qualities enter the market. The DEJ curve shows aggregate quality in the competitive case, with the DE and EJ segments corresponding to each expression in Eq. (11).

5. Conclusions

In this paper we examined the impact of an ad valorem tax on a vertically differentiated commodity. In situations where consumers rank products unanimously at prices reflecting marginal cost, such a tax is likely to reduce market concentration. Starting from a natural monopoly we found a tax level beyond which the ad valorem tax will induce the entry of at least another firm. Our result is a direct implication of the finiteness property which imposes an upper bound independent of the fixed cost level to the number of active firms in the price equilibrium. Thus, once the fixed cost has reached a low enough level to allow the number of firms to attain its upper bound, no further reduction of the fixed cost can affect concentration. Unlike the effect of reductions in the fixed cost, increases in the tax rate may result in a larger number of firms.
For sufficiently low levels of fixed cost, large tax rates may destroy the oligopolistic market structure and induce the entry of a very large number of firms. This will happen when the tax destroys consumer unanimity at prices equal to marginal cost by making the unit-variable-cost-as-function-of-quality schedule too steep relative to the marginal-consumer-willingness-to-pay-for-quality function. In that case each consumer will have his own most suitable quality and if the fixed cost is zero there will be an infinite number of products, one for each taste.

It must be stressed that the effect of changes in the tax rate on market structure is abrupt. At initial low rates continuous increases in the tax rate result in the periodic addition of one more firm in the market. When the tax rate crosses the critical level that destroys consumer unanimity over products, there may be a massive entry of new firms depending on the level of the fixed cost.

These conclusions are easily generalizable for any cost-preference structure that yields consumer unanimity over product ranking at prices reflecting marginal cost. They also hold for other types of taxes. Consider, for instance, a tax on an input that is used more intensively in the production of higher qualities. The exact way such a tax affects the marginal cost schedule depends on differences in the production process of different qualities with respect to the elasticity of substitution away from this input. Nevertheless, our qualitative results still hold provided that, after all the cost-minimizing substitutions have taken place, the after-tax marginal cost schedule is steeper with respect to quality changes.

The above results have policy implications. As shown in the paper, except for corner solutions, a monopolist will underprovide quality. Hence, for low values of fixed cost, first best optimality can be achieved either by an ad-valorem subsidy to the monopolist or by a “large” ad-valorem tax that breaks the finiteness property.

Further, there are cases in which consumption of higher quality generates negative externalities as is the case with the environmental impact of some products. For instance, paper products are traditionally ranked by consumers according to their whiteness, which depends on the intensity of the use of bleach; vegetables are ranked according to their shape and appearance, which depends on the intensity of the use of pesticides and fertilizers; fur coats and other luxury items often require the use of inputs that are near extinction or nearly depleted, etc. Despite the fact that from a consumption point of view a monopoly

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14 When the taxed input is used in fixed proportions to output the input tax is identical to the ad valorem tax treated in the text.

15 Our results hold even if in the pre-tax situation quality improvements do not require a more intensive use of the taxed input, provided that the elasticity of substitution decreases with quality.

16 Crémer and Thisse (1994b) analyses cases where environmental awareness leads consumers to differentiate (otherwise similar) products according to the environmental impact of their production process. Starting from a natural duopoly, the authors arrive independently at conclusions similar to ours concerning the impact of a tax on market structure.
underprovides quality, the policy maker may wish to further lower aggregate quality. The design of a tax on the basis of a given market structure may be misleading. This is so since any increase in the tax rate that reduces market concentration causes also a jump in the level of aggregate quality. Failure to acknowledge the endogeneity of market structure may result in higher aggregate quality and environmental deterioration in the post-tax situation.

Note, finally, that while most aggregate quality targets can be achieved by two (or more) distinct tax rates, the welfare implications of such rates can be very different: in contrast to the monopoly case in which a single quality is produced and consumed by everyone, in the competitive case a whole spectrum of qualities is available. Consumers with high taste for quality consume a higher quality product than under monopoly, whereas those with low taste for quality consume a lower quality product. On the other hand, with convex costs it is costlier to reach a given level of aggregate quality through the production of an array of products than by means of a single quality. A complete welfare analysis involves interpersonal comparisons and lies outside the scope of this paper.

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