The causal relationship between

FT-SE 100 Stock Index Futures Volatility & FT-SE 100 Index Options Implied Volatility

by

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Abstract

The primary objective of this paper is to examine the relationship between historical and implied volatility estimates in order to propose the nature of a model that predicts the future volatility. For this scope, we used non-overlapping clustering of observed futures volatility and implied volatility estimates from options on futures. Three types of implied volatility were used (ATM, NTM (calls), NTM (puts)). Standard deviation of logarithmic returns was used to represent historical volatility. In turn, we utilise Augmented Dickey-Fuller (ADF) unit root tests in order to check the time series properties of the data before running Granger tests of causality between several types of historical and implied volatility.

Key words: implied volatility, Black model, stationarity, Granger causality

JEL Classification: G15, C22


1. Introduction

After the final collapse of the fixed exchange rates system (1973), the alternative floating exchange rates system has created a huge impact on the price volatility of financial and commodity markets. Derivative products had started to become dominant in international financial markets, as academics and practitioners were trying to find alternative ways to protect commodity or financial products from unfavorable exchange rate movements. For many years, research has been directed toward investigating the accuracy of volatility forecasts obtained from various econometric models (Brailsford and Faff, 1996) using data from historical volatility of the underlying asset. Recently, implied volatility obtained from stock index options has become both a dominant and an accurate measure for actual volatility (Chiras and Manaster, 1978; Gwilym and Buckle, 1999; Christensen and Hansen, 2002). In order to find the relationship between historical and implied volatility, an examination of the level and stationarity of volatility over time becomes a prerequisite.

Although volatility forecasts have many practical applications such as the use in the analysis of market timing decisions, aid with portfolio selection and the provision of estimates of variance for use in option pricing models, in our study we are not trying to find the predictive ability of various models but the intertemporal relationship between the forecasts of different nature.

2. Types of Volatility

The term volatility, technically, refers to a statistical measure of dispersion around a mean value or under a theoretical aspect of view it is the changeability or randomness of the underlying asset (Schwert, 1990). Because of the central role that volatility plays in derivatives valuation, a
substantial literature is devoted to the specification and modeling of volatility as it is appeared to be highly unpredictable (Abken and Nandi, 1996). Although volatility can take several forms, such as historical (backward-looking), forecasting or expected (forward-looking) or implied (reflected in an option market price), one type of volatility is that every researcher would like to know: the actual ‘future’ realized volatility. In theory, this is the volatility measure we use when we speak of the volatility input into Black (1976) model. Because volatility is difficult or impossible to measure directly, historical volatility is often used as an estimate of volatility or as a starting point for predicting volatility. On the other hand, implied volatility makes an estimation of volatility of the underlying asset for the aggregate period, from the time of the observation up to the expiration date.

2.1. Historical or Backward-looking Volatility

Although there is a vast number of researchers who have tested different approximations of historical volatility, the literature can be divided in (a) volatility estimated by the standard deviation computed by continuously compounded or logarithmic asset returns (Shastri and Tandon, 1986; Cho and Frees, 1988; Ritchken, 1996) and (b) volatility estimated by the standard deviation of discrete time or arithmetic asset returns (Poterba and Summers, 1986; Schwert, 1990; Brailsford and Faff, 1996). Generally, the models of historical volatility make simplified assumptions, such as the stationarity of the mean of returns, whereas, the mean changes over time. Although, ceteris paribus, more data generally lead to more accuracy, data for volatility that are too old may not be relevant for predicting the future.
2.2. Implied volatility

Generally speaking, historical volatility is associated with the underlying asset. There is, however, a different kind of volatility, called implied volatility (Latane and Rendleman, 1976\textsuperscript{12}), which is associated with an option's market value rather than with the market value of the underlying asset. Obtaining implied volatility requires the use of Black option pricing model. It is the volatility we must feed into our theoretical pricing model to yield a theoretical value identical to the price of the option in the market (Natenberg, 1994\textsuperscript{13}). Knowing the market price of an index option, we have to utilise the Black model in order to derive implied volatility. The Black's pricing model for European options assumes that implied volatility is invariable. Moving beyond the constant volatility assumption implied volatility is modelling according to OLS, ARCH and GARCH specifications.

If pricing model held exactly, then options with different strike prices and expiration dates would be priced in a way to yield the same implied volatility. One cannot ignore the phenomenon of different implied volatilities for different strike prices, called in the literature as "volatility smile" (Natenberg, 1994\textsuperscript{13}; Abken and Nandi, 1996\textsuperscript{6}). Furthermore, we cannot rule out the fact that the implied volatilities from the calls and puts may be different. There is reason to believe that the implied volatility of the put option is higher than that of the call because put is a natural hedging instrument and investors use it as an instrument of insurance (Ncube, 1996\textsuperscript{14}).

2.3. Forward-looking volatility

While the historical volatility of an asset return is readily computed from observed asset returns and the implied volatility is computed from observed option prices, there may be inaccurate
estimators of the volatility expected to prevail over a period of time. Instead of the implicit assumption of stability, many authors have indicated that the volatility is time-varying (see Bollerslev, Chou and Kroner, 1992¹⁵).

The simplest relaxation of the constant volatility assumption, allows volatility to depend on its past information in such a way that future volatility can be perfectly predicted from its history. Abken and Nandi⁶ (1996) and Brailsford and Faff³ (1996) suggested a method in which the variance of asset returns is described by the following equation:

\[
\sigma_{t+1}^2 = \alpha + \beta \sigma_t^2
\]  

(1)

The above equation models the volatility in such a way that the future volatility depends on a constant and a constant proportion of the last period’s volatility (Abken and Nandi, 1996⁶). Moving beyond, many authors allow the future volatility of the underlying asset to depend on either the past volatility of the underlying asset or the implied volatility derived from the relevant option contract (Canina and Figlewski, 1993¹⁶, Christensen and Prabhala, 1998⁴ii). But the major question remains: "which type of volatility leads the other?"

2.4. Comparison between historical-based and implied-based volatility estimates

Several authors (Schmalensee and Trippi, 1978¹⁷; Natenberg, 1994¹³; Christensen and Prabhala, 1998⁴ii; Hansen, 2001¹⁸) found that implied volatility estimates are superior to the historical based volatility estimates of any kind at predicting future volatility values, while others found evidence that the historical volatility is better predictor than implied volatility (Lamoureux and Lastrapes, 1993¹⁹; Canina and Figlewski, 1993¹⁶). Furthermore, Jorion²⁰ (1995) claimed that implied
standard deviations are biased forecasts of future volatility, and were found to be worse estimators than historical based volatility estimators. However, this phenomenon can be given two possible explanations: either the test procedure is faulty, or option markets are inefficient. In order to have a more integrated view about historical and implied volatility, it is preferable to examine the relation between them. Implied volatility can be thought as a ‘consensus’ volatility among all market participants with respect to the expected amount of underlying price fluctuation over the remaining life of the option (Natenberg, 1994). On the other hand, historical volatility is an actual variable and it is essential for market participants to use it for pricing options. It is widely known that when historical volatility rises, implied volatility of all options is likely to rise. From the market participants’ point of view, some traders may change their volatility forecast in response to changing historical volatility. Consequently, it is logical to assume that the whole market will also change its consensus volatility in response to changing historical volatility. Implied volatility is affected by several other factors, apart from historical volatility, such as government announcements, speculative trading activities and various events that are likely to appear in the future. Isolating historical volatility effects, we can support that when the market becomes more volatile, implied volatility can be expected to rise.

3. Methodology, Data & Empirical Issues

3.1. Volatility Measures

The original options pricing model was developed by Black and Scholes (1973). Since then, a number of authors have been extended the original approach to apply to special cases (Black, 1976; Garman and Kohlhagen, 1983; Grabbe, 1986). Because of the fact that in our analysis we deal with LIFFE FT-SE 100 Index Option contract based on the respective future contract,
the appropriate valuation model would be that of Black\textsuperscript{7i} (1976). The aforementioned model has the following form:

\[ C = e^{-rt}[FN(d_1) - KN(d_2)] \]

where

\[ d_2 = d_1 - \sigma \sqrt{t} \]

and

\[ d_1 = \frac{\ln \left( \frac{F}{K} \right) + \left( \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}} \]

where \( C \) denotes the price of a futures call option; \( F \) denotes the underlying future price; \( K \) denotes the futures option exercise price; \( t \) is the time to expiry in years; \( r \) is the risk-less rate of return; \( N(.) \) is the standard normal distribution function; \( \sigma \) is the standard deviation of returns on the futures contract.

Our empirical analysis was based on data concerning daily closing prices for FTSE 100 ATM (at-the-money) and NTM (near-the-money) Call and Put Index Options (European Style) along with daily closing prices for Futures Contracts for a period from 7/1997 to 11/2001, provided by the London International Financial Futures Exchange (LIFFE). We should point out the problem of asynchronous trading due to non-simultaneously trading along with the different trading hours for option and futures markets. Having in mind that even if market makers were to price options according to the established Black model, transactions costs, low liquidity and non-synchronous trading would cause implied volatilities to lie within a wide range of values. Thus, in order to avoid an unjustified weighting scheme we used ATM Call Implied Volatility estimates provided
by LIFFE. For near-the-money implied volatilities derivation we used Bharadia et al.\textsuperscript{23} methodology.

We used the approach used by Christensen and Pradhal\textsuperscript{a} (2002) for filtering the data. In order to trace the call options for deriving the corresponding implied volatility values we choose options that are locating on the Wednesday after the expiration Friday. Following the above procedure, we have 53 observations to deal with.

As far as historical volatility concerns, we create 53 monthly volatility clusters using the remaining trading days (from Wednesday after the expiration Friday up to next the expiration Friday). In order to estimate monthly historical volatility we used the standard deviation of lognormal returns on the underlying asset (FT-SE 100 Future):

$$\sigma_t = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - m)^2}$$

(5)

where \(m\) denotes the logarithmic mean return over a month and \(x_i\) the individual observation of logarithmic return. The annualized volatility is given by:

$$\sigma_t \sqrt{12}$$

(6)

### 3.2. Stationarity & Causality of Volatility Measures

Engle and Granger\textsuperscript{24} (1987) have shown that many time series variables are non-stationary or integrated of order 1. There are two types of time series variables considered in this paper: historical and implied volatility. Consequently, the implied volatility measure is divided in three categories: at-the-money implied volatility, near-the-money call implied volatility and near-the-money put implied volatility. Thus, we will involve with these four variables.
In order to avoid spurious regression situation the variables in a regression model must be stationary or cointegrated. So, we test the aforementioned variables for unit roots. For this purpose, the Augmented Dickey – Fuller (ADF) test is used. The test for the order of integration, it is common practice to run the ADF test, which involves estimating the following equation:

\[ \Delta Y_t = \alpha_0 + \alpha_1 t + \alpha_2 Y_{t-1} + \sum_{j=1}^{p} \beta_j \Delta Y_{t-j} + \varepsilon_t \]  

(7)

where \( \Delta \) is the difference operator and \( j=1,2,3,\ldots,p \) is the number of lag terms.

The ADF regression tests for the existence of unit root of \( Y_t \), namely in the logarithm of all model variables at time \( t \). The variable \( \Delta Y_{t-j} \) expresses the first differences with \( p \) lags and final \( \varepsilon_t \) is the variable that adjusts the errors of autocorrelation. The coefficients \( \alpha_0, \alpha_1, \alpha_2, \) and \( \beta_j \) are being estimated. The null and the alternative hypothesis for the existence of unit root in variable \( Y_t \) is:

\[ H_0 : \alpha_2 = 0 \]

\[ H_1 : \alpha_2 < 0 \]

If \( \alpha_2 \) is negative and significantly different from zero, the null hypothesis of nonstationarity is rejected. The constant and the trend terms are retained only if they are significantly different from zero. The optimal number of lags is determined by minimizing the Akaike Information Criterion (AIC).
Table 1. Test of stationarity for volatility
Augmented Dickey Fuller (ADF) Unit Root Test

<table>
<thead>
<tr>
<th>Lags</th>
<th>HIST</th>
<th>IATMV</th>
<th>INTMC</th>
<th>INTMPV</th>
<th>HIST</th>
<th>IATMV</th>
<th>INTMC</th>
<th>INTMPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4.5885*</td>
<td>-4.2945*</td>
<td>-4.2595*</td>
<td>-4.5720*</td>
<td>-4.3378*</td>
<td>-4.3000*</td>
<td>-4.3010*</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-4.2303*</td>
<td>-3.9782*</td>
<td>-3.9505*</td>
<td>-4.2198*</td>
<td>-4.0324*</td>
<td>-4.0012*</td>
<td>-3.9978*</td>
<td></td>
</tr>
</tbody>
</table>

Critical Values at 95%: -2.9167, -3.4952

Akaike Information Criterion (AIC) defines the optimal number of lags, for both cases for all variables, at 0
*: denotes significance in 95% level

It is seen (Table 1) that all variables are integrated of order zero (I(0)). Since all four variables are stationary, integrated of order zero (I(0)) we can estimate a regression with the existed data.

Now, we investigate the direction of causality between historical and implied volatility.

Following Granger26 (1969), a financial time series $Y_t$ is said to be 'Granger caused' by another series $X_t$ if the information in the past and present values of $X_t$ helps to improve the forecasts of the $Y_t$. The conventional Granger causality test involves specifying a bivariate pth order VAR model as follows:

$$Y_t = \mu + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{j=1}^{p} \beta_j X_{t-j} + u_t,$$

$$X_t = \mu^* + \sum_{i=1}^{p} \nu_i Y_{t-i} + \sum_{j=1}^{p} \xi_j X_{t-j} + u_t^*$$

where $\mu$ and $\mu^*$ are constant terms, $u_t$ and $u_t^*$ are error terms.
The null hypothesis that $X_t$ does not Granger cause $Y_t$ and the alternative hypothesis that $X_t$ does Granger cause $Y_t$ amount to testing:

$$H_0^{(a)}: \beta_1 = \beta_2 = \ldots = \beta_p = 0$$

$$H_1^{(a)}: \text{At least one } \beta_i \neq 0$$

Similarly, the null hypothesis that $Y_t$ does not Granger cause $X_t$ and the alternative hypothesis that $Y_t$ does Granger cause $X_t$ amount to testing:

$$H_0^{(b)}: \nu_1 = \nu_2 = \ldots = \nu_p$$

$$H_1^{(b)}: \text{At least one } \nu_i \neq 0$$

<table>
<thead>
<tr>
<th>Table 2. Causality Analysis</th>
<th>Pairwise Granger Causality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>H0</td>
<td>Probability Value</td>
</tr>
<tr>
<td>Implied At-The-Money Volatility does not Granger Cause Historical Volatility</td>
<td>0.36243</td>
</tr>
<tr>
<td>Historical Volatility does not Granger Cause Implied At-The-Money Volatility</td>
<td>0.00024</td>
</tr>
<tr>
<td>Implied Near-The-Money Call Volatility does not Granger Cause Historical Volatility</td>
<td>0.35366</td>
</tr>
<tr>
<td>Historical Volatility does not Granger Cause Implied Near-The-Money Call Volatility</td>
<td>0.00026</td>
</tr>
<tr>
<td>Implied Near-The-Money Put Volatility does not Granger Cause Historical Volatility</td>
<td>0.34850</td>
</tr>
<tr>
<td>Historical Volatility does not Granger Cause Implied Near-The-Money Put Volatility</td>
<td>0.00026</td>
</tr>
</tbody>
</table>
In equation (8), if we reject $H_0^{(a)}$, we could conclude that $X_t$ leads to $Y_t$. Similarly, in equation (9), if we reject $H_0^{(b)}$, we could conclude that $Y_t$ leads to $X_t$.

In our analysis, we used Akaike Information Criterion (AIC) to determine the lag length of $p$. The results show that the optimum lag length is equal to 2 in each case. The probability values corresponding to the causality tests are presented in Table 2.

The results demonstrate that the hypothesis that historical volatility Granger caused by implied volatility is not supported by the data. However, we found that the hypothesis that the historical volatility-led implied volatilities, of any type (at-the-money implied volatility, near-the-money call implied volatility and near-the-money put implied volatility), are existed is strongly supported by the data.

The aforementioned findings have the following financial interpretation: Market participants, using the knowledge of historical volatility, formulate the prices of the option market through their trading activities. In turn, those prices have an impact to all types of implied volatility. The impact could be obvious because there are instantaneous trading activities that are taking place throughout a specific trading day.

4. Summary & Conclusions

After examining the level of stationarity we reject the hypothesis of non-stationarity at 95% confidence level for all variables. Thus, our variables are considered to be stationary at their levels ($I(0)$).
Following that, our study extends on conventional measures of contagion by directly investigating causality patterns between realized and implied volatility by using the Granger-causality methodology. In cases that a type of implied volatility was used as dependent variable in the Granger OLS regression, non-causality hypothesis is rejected at a high confidence level indicating that recent historical volatility adds more information about future implied volatility than past implied volatility does alone.

The issue of whether implied volatility is a better predictor of the future realized volatility than historical volatility is examined by conducting further Granger causality tests. According to these, we found that implied volatility contains no more information regarding future realized volatility that is not already contained in recent historical volatility.

Even though in all cases historical volatility provides additional information regarding the prediction of the future implied volatility, we cannot conclude the quantity of the influence and the trade-off between the historical volatility and the current implied volatility. The above leaves an open-window for a future researcher to examine the exact impact of both categories of volatility on the future implied volatility. The above may be useful for market participants to exploit 'volatility trading' devises.
REFERENCES


