Reaching extreme events with conditional and unconditional models

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Abstract: - Classical models often fail to predict values arising from crises events, partly, because they are based on the assumption that financial return series follow normal distribution. On the other hand, recent empirical studies have shown that financial data exhibit non-normality and heavy-tails. Historical Simulation is able to overcome obstacles arising from assumptions about the shape of the risk factors’ distribution. Moreover, Extreme Value Theory (EVT) is becoming a key element for modern risk managers and regulators, in the analysis of heavy-tailed data and the estimation of extreme quantiles as it derives the laws governing rare events without implying any statistical distribution assumptions for the dataset. Choosing from the restricted spectrum of historical simulation and EVT the key issue is whether to choose a conditional or unconditional model.

Key-Words: - Extreme Value Theory, GARCH models, conditionality, risk management, capital requirements

1 Introduction

The recent history exhibits a broad set of examples on financial disasters but their consequences on market participants varied from default events to sharp decrease in profitability. Learning from these lessons, regulators are trying to formulate a sound and stable financial system. In order to achieve their objectives they imposed conservative capital requirements by the use of the standard approach (Basle, 1996).

On the other hand, financial institutions, being compliant with rules imposed by regulatory agencies, try to satisfy shareholders’ appetite for continuously increasing profits, without undertaking extreme risks.

Both market participants and regulators are under a continuous discussion, striving to find a bilaterally accepted theoretical common ground. Within this consensus, quantitative probabilistic risk management techniques (Value-at-Risk type approaches) were implemented as internal models, widely accepted by both as the most suitable way for measuring individual market risk exposures. VaR traditionally involves three methods developed throughout the last decade. Variance - Covariance method is based on the assumption that the joint distribution of risk factors can be approximated by an a priori theoretical distribution (usually Normal) which tends to underestimate the tails of the return distribution. Historical Simulation is subject to event risk because of the lack of sufficient data set (Longin, 2000). Finally, Monte Carlo simulation is subject to model error
risk because it is often based on models selected from a limited spectrum of models.

Empirical literature (Raatikainen, 2002) is moving beyond the aforementioned VaR models towards Extreme Value Theory (EVT) models. EVT refers to two alternative classes of models for extreme values: the Block Maxima approach (BM) and the Peaks-Over-Threshold (POT) approach.

In our paper we will focus on the semi-parametric POT, structured around the Hill estimator (Hill, 1975) in an attempt to examine representative European Union market data in order to select the best VaR model from (un)conditional EVT and historical simulation approaches.

Finally, we examine the effect of each method on the calculation of capital requirements for banking sector’s trading book, under the new Basel Accord and, possible dilemmas that regulators will face trying to impose the most efficient method.

2 Theoretical Background

2.1 Historical Simulation

Historical simulation-based methods have become increasingly popular because of their easiness to implement, their ability to capture second-order risks through various means, and their independence of the covariance matrix of the risk factors. But the most important features are that different distribution assumptions can be adapted for different risk factors (e.g. fat tails, leptokurtosis, stochastic volatility, mean-reversion, etc.) and that valuation of complex instruments can be carried out.

2.1.1 Unconditional approach

Unconditional historical based estimation of VaR or widely known unconditional historical simulation VaR (UHS) is the simplest way to compute, in terms of minimal computational power and parametric assumptions, the market risk of financial instruments. These are, mainly, the motivations that force many financial institutions to use the aforementioned method.

The strict stationarity assumption underlying UHS implies that the distribution of the risk factors realized in the past can be used to simulate the distribution of changes in value of the present portfolio. Therefore, the length of data available has a direct effect on the ability to predict extreme economic recessions in the future.

The VaR estimate for the t+1 day, given a probability level q, is given by the empirical quantile \( Q_{q} \) of a window of \( n \) independently and identically distributed (iid) observations up to date \( t \), that is

\[
\text{VaR}_{t+1} = \hat{Q}_{q}(r_t, r_{t-1}, ..., r_{t-n+1})
\]  

(1)

For example, examining a window of \( n=5000 \) observations, the 1% VaR estimate is the 50th negative value of the sample ascending order statistic. In this case 49 extreme events are opting out the estimation procedure.

The iid assumption implies that the model assigns an equal probability weight of \( 1/n \) to each observation (Pritsker, 2001). However, volatility of asset returns tends to appear in clusters, that is, high volatility observations and low volatility observations are grouped together (Bollerslev, 1986). Boudoukh, Richardson, and Whitelaw (1998), deviated the problem by assigning higher weights to the recent past observations than the distant past observations, formulating a generalised version of the historical simulation.

2.1.2 B-A.G.V. Filter specification

Adapting the assumption of normality, variance-covariance methods (not examined here) attempt to capture the conditional heteroskedasticity of the asset returns by estimating variance-covariance matrices at every point in time.

By contrast, unconditional historical simulation method does not assume any return distribution but, typically, it is unable to capture conditional heteroskedasticity.

In this context, Barone-Adesi, Giannopoulous, and Vosper (1999), introduced a variant of the historical simulation methodology referred as filtered historical simulation (FHS). This approach introduces an innovation that captures both conditional heteroskedasticity and non-normality of asset returns within a unique model, improving the performance of classical variance-covariance and historical simulation methods that are currently in use.

Assuming lognormal asset returns as

\[
r_t = \ln \left( \frac{X_t}{X_{t-1}} \right)
\]  

(2)
where $X_t$ is the index value at time $t$ and $X_{t-1}$ is the lagged index value.

B-A.G.V. methodology proposes the estimation of an ARMA(1,1)-GARCH(1,1) without excluding alternative models

\[ r_t = a_0 + a_1 r_{t-1} + a_2 \varepsilon_{t-1} + \varepsilon_t \quad (3) \]

\[ \sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 \quad (4) \]

with

\[ \varepsilon_t \sim N(0, \sigma_t^2) \]

Although forecasting for multiple days ahead is feasible we focused on next day predictions. The VaR is estimated from the following sequence of equations after obtaining the future return distribution from 1000 simulations.

\[ \varepsilon_t = \frac{\varepsilon_t}{\sigma_t} \quad (5) \]

\[ \sigma_{t+1}^2 = \beta_0 + \beta_1 \varepsilon_t^2 + \beta_2 \sigma_t^2 \quad (6) \]

\[ z_{t+1} = \varepsilon_t \sigma_{t+1} \quad (7) \]

\[ r_{t+1} = a_0 + a_1 r_t + a_2 z_t + z_{t+1} \quad (8) \]

For the remaining $\sigma_{t+1}$, $\varepsilon_t$ term in the first equation of the sequence is substituted by $z_{t+1}$. Eventually, a quantile is derived form the forecasted distribution of returns. The s-day ahead VaR is calculated by subtracting the aforementioned quantile from t-day actual value.

A criticism on this method is that when a VaR model is examined using real data, the ability to understand the properties of the VaR model is obfuscated by the simultaneous occurrence of other types of model errors including errors in pricing, errors in GARCH models, and other potential flaws in the VaR methodology (Pritsker, 2001).

### 2.2 Extreme Value Theory

Extreme Value Theorem or Fisher-Tippett Theorem and its variants (Fisher and Tippett, 1928), originally applied to model rare phenomena came from hydrology and climatology. Albeit being among the oldest theories in statistical engineering, only recently have systematically being employed to explain extreme, possibly out of sample, behaviors came from the field of insurance and finance (McNeil, 1999; Kellezi and Gilli, 2000; Blum and Dacorogna, 2003).

#### 2.2.1 Unconditional approach

**Semi-parametric POT:** The financial risk management, in general, is focused on heavy-tailed distributions in the maximum domain of attraction of GEV, that is, $\xi > 0$, for which

\[ 1 - F(x) = x^{-\alpha} L(x) \quad (9) \]

Except for statistical properties, the term $L(x)$ can be eliminated from the statistical estimates (Blum and Dacorogna, 2003), without imposing any significant bias.

In such cases, Hill (1975) proposed an estimator of $\xi$ that was meant to be used in cases that $\xi > 0$ (Frechet class models). First original $X_i$ data, that are assumed to be independently identically distributed (i.i.d.), are arranged in a descending order, as, $X_{(1)} \geq X_{(2)} \geq \ldots \geq X_{(n)}$. In turn, the tail index $\xi$ is

\[ \hat{\xi}_{n,m} = \frac{1}{m-1} \sum_{i=1}^{m-1} \ln X_{(i)} - \ln X_{(m)} \quad (10) \]

Index $m$ denotes the $m^{th}$ threshold element or, otherwise stated, the cut-off point of the descending ordered sample, over which extreme values are realized. The difficulty in threshold determination is that a high threshold provides us with little information about the behavior of extremes, while a low threshold introduces some of the average observations from the dataset distribution, increasing the bias incorporated in the estimation. Dacorogna et al (2001) and Blum and Dacorogna (2003) defined $m$ as the square root of the number (n) of observations and found this a fairly good approximation of the true value, retaining the trade off between bias and availability of data.

In order to estimate the extreme quantile $Q_q$, without bias, the formula (11) proposed by Dacorogna et al (2001) should be applied on large samples. For a specified probability level q extreme quantile is given by

\[ \hat{Q}_q = X_{(m)} \left( \frac{m}{nq} \right)^{\frac{1}{\hat{\xi}}} \]

#### 2.2.2 Conditional approach


**Step 1:** They fit a GARCH-type model to the log-return data by quasi-maximum likelihood. That is,
maximize the log-likelihood function of the sample assuming normal innovations.

**Step 2**: It is assumed that the standardized residuals computed in Step 1 are realizations of a white noise. In turn, in order to estimate the tails of these innovations, EVT is used. Finally, the quantile of the innovations are calculated for a given level of $q$.

Let the following equation represent the behavior of log-negative returns

$$ r_t = a_0 + a_1 r_{t-1} + \varepsilon_t $$  \hspace{1cm} (12)

where $a_0$ and $a_1$ are parameters to be estimated, $r_{t-1}$ is the lagged log-return and $\varepsilon_t$ indicates the residua series.

If we suppose that the conditional variance $\sigma^2_t$ of $\varepsilon_t$ follows a GARCH(1,1) process, this is generated by equation (4)

The equation (4) is estimated by maximizing the log-likelihood function of a sample of $n$ observations.

The step 1 ends by calculating estimates of the conditional mean and variance for 1-day ahead forecasts

$$ \mu_{t+1} = \hat{\mu}_0 + \hat{\alpha}_1 r_t $$  \hspace{1cm} (13)

$$ \sigma^2_{t+1} = \hat{\beta}_0 + \hat{\beta}_1 \hat{\varepsilon}_t^2 + \hat{\beta}_2 \sigma_t^2 $$  \hspace{1cm} (14)

where $\hat{\varepsilon}_t = r_t - \hat{\mu}_t$

In the present paper the one day ahead forecast for VaR($e$) is given by applying equation (11) on the negative standardized residuals. Consequently, one-day ahead forecast of the VaR$_q$ is given by

$$ VaR_q = \hat{a}_0 + \hat{a}_1 r_{t-1} + \hat{\sigma}_{t+1} VaR(e)_q $$  \hspace{1cm} (15)

3 **Data - Methodological Issues**

In this paper we examined the behavior of four (4) European Union state countries’ stock exchange indices, namely ASE-G (Greece), CAC-40 (France), DAX (Germany) and FTSE-100 (UK).

The analysis was based on data provided by Bloomberg Professional online contributor. The data we have taken into consideration are daily observations ranging from 03/01/1984 to 06/11/2003 or 5020 observations for the FTSE-100 Index, from 09/07/1987 to 06/11/2003 or 4096 observations for the CAC-40 Index, from 21/06/1976 to 06/11/2003 or 6874 observations for the DAX Index and, from 04/01/1987 to 06/11/2003 or 4195 observations for the ASE-G Index.

The starting point of our methodology is to find the econometric model that best describes the behavior for the entire dataset for each of four stock index returns. The parameter estimates were obtained by the method of quasi-maximum likelihood. That is, the log-likelihood function of the data was constructed by assuming that innovations are conditionally normally distributed.

The estimates of the parameters will be used in the forecasting of one-day conditional VaR for both Historical Simulation and Extreme Value approach.

In turn, we estimated the conditional and unconditional EVT quantile by using estimation procedure of Dacorogna et al. (1995). The unconditional VaR was estimated by the pure return data.

Alternatively, a variant of conditional VaR, introduced by McNeil and Frey (2000), was estimated by, initially, estimating the unconditional VaR of the standardized residuals and then substituting in the AR(1) – GARCH(1,1) equation (16) which was proved to be the best specification for all return series.

4 **Results and Discussion**

Although according to unconditional methods ASE-G appears to be the riskiest index, it is the third riskiest among examined indices according to conditional methods. This proves that the current conditions are indeed posing Greece among the developed countries in market risk terms.

Exactly the opposite view is demonstrated by DAX index, as Germany is currently affected by economic recession. Instead, FTSE-100 is the less risky index under any approximation.

All coefficients estimated for AR(1)-GARCH(1,1) are statistically significant at 99% level, except for the AR(1) term concerning CAC-40 Index that is statistically significant at 95% level and the constant term in the AR(1) regression for ASE-G Index that is statistically insignificant. However, in general, all AR(1)-GARCH(1,1) results are overwhelmingly accepted and were used for both EVT and HS conditional approaches. It can be observed that the MA(1) term was dropped out from the original B-A.G.V. filter specification.
A natural indicator or the expected value of the VaR, given a certain confidence level, is expected to be the value of the order statistic provided by the multiplication of the confidence level with the number of observations included in the population of the asset under discussion. According to that view, unconditional historical simulation VaR is considered to be the natural or the expected VaR.

| Table 2: Daily Value at Risk (%) |
|-------------------------------|-----------------|-----------------|-----------------|
| Prob. | Index | Historical Simulation | EVT |
| | | U | C | U | C |
| 95% | ASE | 2.63 | 1.73 | 3.06 | 1.81 |
| | DAX | 1.95 | 2.33 | 2.19 | 2.25 |
| | FTSE | 1.59 | 1.25 | 1.81 | 1.48 |
| 99% | ASE | 5.58 | 2.65 | 5.39 | 2.75 |
| | CAC | 4.04 | 2.78 | 3.90 | 2.83 |
| | DAX | 3.73 | 4.17 | 3.71 | 3.39 |
| | FTSE | 2.91 | 1.88 | 2.93 | 2.00 |
| 99.9% | ASE | 10.12 | 4.28 | 12.10 | 5.03 |
| | CAC | 7.61 | 3.33 | 7.41 | 4.84 |
| | DAX | 6.91 | 6.28 | 7.85 | 6.08 |
| | FTSE | 5.58 | 2.64 | 5.79 | 3.10 |

U: unconditional
C: conditional

Comparing the results, conditional historical simulation, for 95% and 99% probability level, provides higher values than the unconditional historical simulation only for DAX Index. For the highest probability level examined (99.9%) unconditional forecasts are higher for all stock exchanges.

Being in line with the above results, conditional Extreme Values Theory forecasts yields lower values for all probability levels for all indices, except for DAX Index for 95% level.

In general, conditional models yield lower estimates for VaR. That was happened because conditional models take into account the current conditions in the economy expressed by the current clustered volatility. Instead, unconditional models capture extreme events that have appeared certain times in the price history.

In the above context, it is apparent that unconditional models are suitable to carry out stress testing experiments, while conditional models are properly suited for on-going daily VaR estimations. In other words, the former class of models is directed for use by regulators for stress testing the entire credit system; while, the latter class of models seems to be suitable for practitioners.

An alternative unconditional historical simulation approach, that diminishes the influence of far distant past events, would be the use of a smaller time-moving sample of fixed length involved in the estimation of VaR. Once this approach takes into consideration the current and near past events, conceivably, it is a better on-going VaR estimator compared with large sample based historical simulation.

In contrast, we cannot use time-moving sample technique in order to estimate unconditional EVT models because as the sample size decreases the bias of the tail index estimator increases. Thus, unconditional semi-parametric EVT model can be considered as a genuine stress-testing model.

Looking for links between historical simulation and EVT approaches it can be seen that HS produces systematically lower values for one-day VaR forecasts, apart from the unconditional class of models under which the image was inverted.

The Basel Committee (1996) allows banks to consider price shocks equivalent to a short holding period such as a day, but it recommends a holding period of ten (10) days.

Thus, banks are implicitly encouraged to convert one-day VaR to multiple-days VaR. The most popular methodology is to multiply the original one-day estimate by the root of ten (“root of ten law”) that obviously will yield very high values.

In contrast to that view, Danielsson and De Vries (2000) suggest that should be done by the scaling factor $T^{1/2}$ (T is the horizon length expressed in days) for original estimates made using EVT. This eventually leads to lower multi-day VaRs than would be obtained from the normal rule (Danielsson et al, 1998).
5 Conclusion
The objective of the paper is to identify the method of VaR estimation that well balances the tendency of regulators for conservatism and the need of financial institutions of having their shareholders satisfied.

Our results show that variants of VaR can be manipulated in various ways. First, regulators are inclined to use full sample-based unconditional historical simulation and EVT approaches, as they yield the highest VaR values, in order to test the vulnerability of the credit system. Second, there is no unique recipe for both regulators and supervisors in applying a particular model. Third, we cannot make a clear statement recognizing the better model because different models are constructed for different purposes. However, we can explicitly observe that unconditional specifications provide saver predictions, albeit we are not sure about their accuracy.

In order to test their accuracy, the future researcher should be focus on dynamic back-testing techniques. Dynamic back-testing should be carried out in order to see the true effectiveness of VaR systems in economic terms, instead of applying those systems from a subjective aspect of view.

Another issue for further research is to test the accuracy of the scaling factor proposed by Basel Committee, by comparing the scaled one-day ahead VaRs (by the square root of ten) with the original ten-day ahead VaRs based on ten-day period returns.

References: